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Covariance Prediction in Large Portfolio Allocation

Carlos Trucíos [1,*], Mauricio Zevallos [2], Luiz K. Hotta [2] and André A. P. Santos [3,4]

1 São Paulo School of Economics, FGV, São Paulo 01332-000, Brazil
2 Department of Statistics, University of Campinas, Campinas 13083-859, Brazil; amadeus@ime.unicamp.br (M.Z.); hotta@ime.unicamp.br (L.K.H.)
3 UC3M-Santander Big Data Institute, Universidad Carlos III de Madrid, Getafe 28903, Spain; andre.portela@ufsc.br
4 Department of Economics, Universidade Federal de Santa Catarina, Florianópolis 88040-970, Brazil
* Correspondence: ctrucios@gmail.com

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Abstract: Many financial decisions, such as portfolio allocation, risk management, option pricing and hedge strategies, are based on forecasts of the conditional variances, covariances and correlations of financial returns. The paper shows an empirical comparison of several methods to predict one-step-ahead conditional covariance matrices. These matrices are used as inputs to obtain out-of-sample minimum variance portfolios based on stocks belonging to the S&P500 index from 2000 to 2017 and sub-periods. The analysis is done through several metrics, including standard deviation, turnover, net average return, information ratio and Sortino's ratio. We find that no method is the best in all scenarios and the performance depends on the criterion, the period of analysis and the rebalancing strategy.

Keywords: Minimum variance portfolio; risk; shrinkage; S&P 500

JEL Classification: C13; C53; C58; G11

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1. Introduction

Forecasting returns, volatilities and conditional correlations has attracted the interest of researchers and practitioners in finance since these factors are crucial, for example, in portfolio allocation, risk management, option pricing and hedging strategies; see, for instance, Engle (2009), Hlouskova et al. (2009) and Boudt et al. (2013) for some references.

A well-known stylised fact in multivariate time series of financial returns is that not only conditional variances but also conditional covariances and correlations evolve over time. To describe this evolution, several methods have been proposed in the literature. In general, these methods involve different ways to circumvent the issue of dimensionality. The treatment of this problem is vital for the estimation of large portfolios (composed of hundreds or thousands of assets). As noted by Engle et al. (2017), when dealing with portfolios composed of a thousand time series, many multivariate GARCH models present unsatisfactory performance or computational problems in their estimation. For some multivariate GARCH models, estimation problems arise even for smaller dimensions; see, for instance, Laurent et al. (2012), Caporin and McAleer (2014), Caporin and Paruolo (2015) and de Almeida et al. (2018).

Our empirical application is based on an investor who adopts the minimum variance criterion in order to decide on portfolio allocations. A very large body of literature in portfolio optimization considers this particular policy; see, for instance, Clarke et al. (2011 2006) for extensive practitioner-oriented studies on the performance and composition of minimum variance portfolios. This policy can be seen as a particular case of the traditional mean-variance optimisation.

The mean-variance problem, however, is known to be very sensitive to estimation of the mean returns (Frahm 2010; Jagannathan and Ma 2003).[1] Very often, the estimation error in the mean returns degrades the overall portfolio performance and introduces an undesirable level of portfolio turnover. In fact, existing evidence suggests that the performance of optimal portfolios that do not rely on estimated mean returns is usually better, see DeMiguel et al. (2009).

To obtain the minimum variance portfolio, the key input is the estimate of the conditional covariance matrix. As far as we known, there are few works in the literature comparing the estimation of this matrix for large portfolios, with Creal et al. (2011), Hafner and Reznikova (2012), Engle et al. (2017), Nakagawa et al. (2018) and Moura and Santos (2018) being especially relevant. Given the myriad of models and methods in the literature to estimate the covariance matrix, empirical studies about the comparison of estimates in large portfolios are most welcome.

The paper is intended to assess the performance of several methods to predict one-step-ahead conditional covariance matrices in large portfolios. This is done empirically, by comparing the out-of-sample performance of minimum variance portfolios based on S&P500 stocks traded from 2 January 2000 to 30 November 2017, using measures such as average (AV), standard deviation (SD), information ratio (IR), Sortino's ratio (SR) (Sortino and van der Meer 1991), turnover (TO) and average portfolio net of transaction cost (AV^{net}). Since not all stocks of the index were traded during the whole period, we consider portfolios of dimension $N = 174$ stocks. To assess the robustness of the results, we also the analyse three sub-periods: the pre-crisis period (January 2004 to December 2007), the subprime crisis period (January 2008 to June 2009), and the post-crisis period (July 2009 to November 2017).

We consider several attractive methods and models including recent proposals used by practitioners and academics to predict one-step-ahead conditional covariance matrices. They are selected mainly because they use different approaches to overcome the issue of dimensionality problem. Specifically, the paper compares the DCC model as used in Engle et al. (2017), the DECO model of Engle and Kelly (2012), the OGARCH model of Alexander and Chibumba (1996), the RiskMetrics 1994 and the RiskMetrics 2006 (Zumbach 2007) methods, the generalised principal volatility components analysis (GPVC) proposed by Li et al. (2016) as a generalisation of the procedure of Hu and Tsay (2014), and we also apply the robust version of the GPVC method proposed by Trucíos et al. (2019). DCC models are estimated using composite likelihood, as advocated in Pakel et al. (2014). In addition, the linear shrinkage (LS) and non-linear shrinkage (NLS) of Ledoit and Wolf (2004a) and Ledoit and Wolf (2012), respectively, are applied on all the previous methods. Therefore, compared to Engle et al. (2017), Hafner and Reznikova (2012) and Nakagawa et al. (2018), the set of competing methods is much bigger and the device of shrinkage is assessed in all the compared methods. We consider a total of 47 methods, including the equal-weighted portfolio strategy. This constitutes the main contribution of the paper.

The rest of the paper is organised as follows: Section 2 presents the methods and models used to predict the one-step-ahead volatility covariance matrix. It also presents the composite likelihood used to estimate the DCC model and the shrinkage method as presented in Pakel et al. (2014). The empirical application is given in Section 3. Section 4 concludes and the list of the estimation methods is in the Appendix A.

2. The Forecast Methods

Denote by $r_{i,t}$, $i = 1, \ldots, N$, $t = 1, \ldots, T$ the return of the i-th asset at time t, where N is the number of assets under consideration to construct the portfolio and T denotes the sample size. For simplicity, consider that $E(r_{i,t}|\mathcal{F}_{t-1}) = 0$, where $\mathcal{F}_{t-1}$ denotes the information available at time $(t-1)$. Let $\mathbf{r}_t = (r_{1,t}, \ldots, r_{N,t})'$; the conditional covariance matrix is defined as $\mathbf{H}_t = \text{Cov}(\mathbf{r}_t|\mathcal{F}_{t-1})$

[1] See Wied et al. (2013) for a test for the presence of structural breaks in minimum variance portfolios

with elements $h_{i,j,t} = \mathrm{Cov}(\mathrm{r_{i,t}}, \mathrm{r_{j,t}}|\mathcal{F}_{\mathrm{t-1}})$. At time $(t-1)$, we are interested in estimating $\mathbf{H}_t$ in order to select a portfolio for the period $(t-1, t]$. In the following we present some methods to estimate it.

2.1. The RiskMetrics Methods

One of the most popular methods used in risk analysis is the RiskMetrics method developed by the RiskMetrics Group at JP Morgan. We call this the RiskMetrics 1994 (RM1994) method. The main feature of the RiskMetrics method is that the predicted volatility is a linear function of the present and past squared returns. Although it has being widely used, it has some problems. In order to overcome some of these problems, the same group developed the RM2006 method. Like the RM1994 method, the RM2006 method is also data-oriented, in the sense that it was calibrated and tested to have good performance with the majority of the target empirical data, and was developed to take into account some of the stylised facts and weaknesses detected in the RM1994 method. We can summarize the main modifications in three types. In the first type, considering that the volatility has a long memory feature, the weights decay logarithmically instead of exponentially, as happens in the RM1994 method. The second is that the weights depend on the forecast horizon. The third is that the conditional distribution of the return is not multivariate Gaussian; the distribution is based on the estimated devolatilised residuals and it can be roughly defined as a Student-t distribution with scale correction. Finally, the return levels are modelled considering the lagged correlation between returns.

2.2. The CCC Model

The constant conditional correlation model (Bollerslev 1990) is one of the simplest MGARCH models to estimate, since basically the variances are modelled independently and the covariances are obtained using the conditional standard deviation and a constant conditional correlation matrix. The conditional covariance matrix $\mathbf{H}_t$ evolves according to:

$$\begin{aligned}
\mathbf{H}_t &= \mathbf{D}_t\mathbf{R}\mathbf{D}_t, & (1)\\
\mathbf{D}_t &= Diag(d_{1,t}, \ldots, d_{N,t}), & (2)\\
\mathbf{R} &= Diag(\mathbf{H})^{-1/2}\mathbf{H}Diag(\mathbf{H})^{-1/2}, & (3)\\
\mathbf{H} &= \mathrm{Cov}(\mathrm{r_t}), & (4)
\end{aligned}$$

with $d_{i,t}^2 = Var(r_{i,t}|\mathcal{F}_{t-1})$ (marginal univariate conditional variances). The advantage of the CCC model is its easy estimation, although, the main disadvantage is the strong assumption that conditional correlations are time-invariant. Engle (2002) extended this idea in a dynamic conditional correlation way, as detailed in the next section.

2.3. The DCC Model

In this section, we describe the scalar DCC model of Engle (2002) as used in Pakel et al. (2014) and Engle et al. (2017), and the composite likelihood. The non-linear shrinkage method, which is also used to estimate the DCC model, is presented in Section 2.8. In the DCC model, the marginal univariate conditional variances $d_{i,t}^2 = Var(r_{i,t}|\mathcal{F}_{t-1})$ are modelled first. Define the devolatilised residuals as $\mathbf{s}_t = (r_{1,t}/d_{1,t}, \ldots, r_{N,t}/d_{N,t})'$. We use the DCC model with correlation targeting as in Engle et al. (2017). The conditional covariance matrix $\mathbf{H}_t$ evolves according to:

$$\begin{aligned}
\mathbf{H}_t &= \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t, & (5)\\
\mathbf{R}_t &= Diag(\mathbf{Q}_t)^{-1/2}\mathbf{Q}_t Diag(\mathbf{Q}_t)^{-1/2}, & (6)\\
\mathbf{Q}_t &= (1-\alpha-\beta)\mathbf{C} + \alpha\mathbf{s}_{t-1}\mathbf{s}_{t-1}^{'} + \beta\mathbf{Q}_{t-1}, & (7)
\end{aligned}$$

where $\mathbf{D}_t$ is a diagonal matrix with the i-th element of the diagonal equal to $d^2_{i,t}$, $C = Corr(\mathbf{r}_t) = Cov(\mathbf{s}_t)$ is the unconditional correlation matrix, and $R_t = Corr(\mathbf{r}_t|\mathcal{F}_{t-1}) = Cov(\mathbf{s}_t|\mathcal{F}_{t-1})$ is the conditional correlation matrix at time t. The parameters α and β are non-negative with $\alpha + \beta < 1$. We have

$$\mathbf{r}_t|\mathcal{F}_{t-1} \sim WS(0, \mathbf{H}_t), \tag{8}$$

where $WS(0, \mathbf{H}_t)$ means a multivariate distribution with mean zero and covariance matrix $\mathbf{H}_t$.

The model is usually estimated in three stages. In each stage, the estimation is conditional on the estimates found in previous stages. The stages are: (1) estimate $\mathbf{D}_t$ usually assuming a GARCH(1,1) model for each $t = 1, \ldots, T$, and evaluate the devolatilised residuals; (2) select an estimator of the correlation target matrix C using the devolatilised residuals; and (3) estimate the parameters α and β. We will comment on stage one in the application section and on stage 2 in Section 2.8. In the third stage, even with only two parameters, one may face estimation problems with a large number of assets because it is necessary to invert the conditional covariance matrix $\mathbf{H}_t$ (for each $t = 1, \ldots, T$). One way to overcome this problem is through the use of the composite (log-)likelihood[2] to compute it. This method was proposed in the 2008 version of Pakel et al. (2014). In the 2014 version, they showed that the estimators of α and β, given by maximizing the composite likelihood, are consistent although not efficient. They evaluate the composite likelihood by summing the likelihood of all contiguous pairs. Thus, there are only $(N-1)$ bivariate terms and for any contiguous pair it is only necessary to invert a matrix of order two. For instance, let $\mathbf{r}^{(i)} = (r_{i,1}, \ldots, r_{i,T})'$, $i = 1, \ldots, N$, i.e., the series of returns of the ith asset, and denote by $l_i(\alpha, \beta; \mathbf{r}^{(i)}, \mathbf{r}^{(i+1)})$ the likelihood of the pair $(\mathbf{r}^{(i)}, \mathbf{r}^{(i+1)})$, $i = 1, \ldots, N-1$, assuming that each pair comes from a bivariate DCC model, defined similarly as the model given by Equations (5–7). Then, the composite likelihood is given by:

$$CL(\alpha, \beta; \mathbf{r}^{(i)}, i = 1, \ldots, N) = \sum_{i=1}^{N-1} l_i(\alpha, \beta; \mathbf{r}^{(i)}, \mathbf{r}^{(i+1)}). \tag{9}$$

Engle et al. (2017) argue that the estimator of the conditional covariance matrix given by the DCC model using composite likelihood in stage three with the estimation of the unconditional correlation matrix using non-linear shrinkage in stage two is robust against model misspecification in large dimensions (N).

2.4. The DECO Model

Engle and Kelly (2012) propose a dynamic equicorrelation (DECO) model as a trade-off between a model which imposes many restrictions in the covariance matrix and a less structured model. They contend that imposing too much structure can lead to an efficient estimation when the restrictions are correct, but can suffer from breakdown in the presence of misspecification. On the other hand, the lack of restrictions may lead to the issue of dimensionality. Considering this trade-off, they propose a model where the cross-correlations between any pair of returns are equal on the same day, but it can vary over time. In addition, as in the CCC and DCC models, the DECO model also assumes that the marginals are modelled by a univariate volatility model. Using the same notation, we have $d^2_{i,t} = Var(r_{i,t}|\mathcal{F}_{t-1})$, and the covariance matrix is written as $\mathbf{H}_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t$ as in Equation (5). The equicorrelation matrix is given by:

$$\mathbf{R}_t = (1 - \rho_t)\mathbf{I}_N + \rho_t\mathbf{J}_N, \tag{10}$$

where ρ_t is the equicorrelation, $\mathbf{I}_N$ denotes the N-dimensional identity matrix and $\mathbf{J}_N$ is the $N \times N$ matrix of ones. According to Engle and Kelly (2012), $\mathbf{R}_t^{-1}$ exist if and only if $\rho_t \neq 1$ and

2 From now on we just call the log-likelihood likelihood.

$\rho_t \neq -1/(N-1)$, and $\mathbf{R}_t$ is positive definite if and only if $\rho_t \in (-1/(N-1), 1)$. The evaluation of the likelihood is easy because we have closed forms for $\mathbf{R}_t^{-1}$ and $\det(\mathbf{R}_t)$, given by:

$$\mathbf{R}_t^{-1} = \frac{1}{1-\rho_t}\mathbf{I}_N - \frac{\rho_t}{(1-\rho_t)(1+[N-1]\rho_t)}\mathbf{J}_N, \tag{11}$$

and

$$\det(\mathbf{R}_t) = (1-\rho_t)^{N-1}\left[1+(N-1)\rho_t\right], \tag{12}$$

respectively. This description of the DECO model corresponds to a single block. The DECO model can also be used considering many blocks, as described in Engle and Kelly (2012).

2.5. The OGARCH Model

Alexander and Chibumba (1996) propose the Orthogonal GARCH (OGARCH) model, a dimension reduction technique to model the conditional covariance matrix. The model intends to simplify the problem of modelling an N-dimensional system into modelling a system of K-dimension orthogonal components where those components are obtained through principal component analysis ($K \leq N$). Since the components are orthogonal, the conditional covariance matrix of the whole system can be obtained as:

$$\mathbf{H}_t = \mathbf{A}\mathbf{D}_t\mathbf{A}' + \mathbf{V}_\epsilon, \tag{13}$$

where $\mathbf{A}$ is an $N \times k$ matrix whose columns are the normalised eigenvectors associated with the unconditional covariance matrix, $\mathbf{D}_t$ is a diagonal matrix whose elements are the conditional variances of the k principal orthogonal components associated with the k largest eigenvalues, and $\mathbf{V}_\epsilon$ is the covariance matrix of the errors that can be ignored. The conditional variances of each component can be modelled by a GARCH-type model.

Alexander and Chibumba (1996) and Alexander (2002) emphasise the importance of using a number of components k much smaller than N. However, Bauwens et al. (2006) and Becker et al. (2015) suggest using $k = N$ to avoid problems related with the inverse of $\mathbf{H}_t$. The OGARCH model with $k = N$ is a particular case of the GO-GARCH model (Van der Weide 2002).

2.6. The Generalised Principal Volatility Components Model

The generalised principal volatility components (GPVC) procedure is a dimension reduction technique recently proposed by Li et al. (2016), which decomposes a series into two groups of volatility components. The first group corresponds to a small number of components with volatility evolving over time while the second one corresponds to components whose volatility is constant over time. The GPVC procedure considers an orthogonal matrix $\mathbf{M} = [\mathbf{A} \,:\, \mathbf{B}]$ and decomposes an N-dimensional vector $\mathbf{y}_t = (y_{1t}, ..., y_{Nt})'$ with $E(\mathbf{y}_t|\mathcal{F}_{t-1}) = 0$ into:

$$\mathbf{y}_t = \mathbf{M}\mathbf{M}'\mathbf{y}_t = (\mathbf{A}\mathbf{A}' + \mathbf{B}\mathbf{B}')\mathbf{y}_t = \mathbf{A}\mathbf{f}_t + \mathbf{ffl}_t, \tag{14}$$

with $\mathbf{f}_t = \mathbf{A}'\mathbf{y}_t$ and $\mathbf{ffl}_t = \mathbf{B}\mathbf{B}'\mathbf{y}_t$. The matrix $\mathbf{M}$ is obtained through the decomposition $\mathbf{G}\mathbf{M} = \mathbf{\Lambda}\mathbf{M}$, where $\mathbf{\Lambda}$ is a diagonal matrix with elements given by the eigenvalues in decreasing order and $\mathbf{M}$ is the associated matrix of normalised eigenvectors. The columns of matrices $\mathbf{A}$ and $\mathbf{B}$ are the eigenvectors associated with the non-zero and zero eigenvalues, respectively, which are obtained from the eigenvalue decomposition of the matrix $\mathbf{G}$. In practice, $\mathbf{G}$ is given by:

$$\mathbf{G} = \sum_{k=1}^{g}\sum_{t=1}^{T}\omega(\mathbf{y}_t)E^2\left[\left(\mathbf{y}_t\mathbf{y}_t' - \mathbf{\Sigma}\right) I(\|\mathbf{y}_{t-k}\| \leq \|\mathbf{y}_t\|)\right], \tag{15}$$

where g is a positive integer that gives the maximum lag order considered, $\omega(\cdot)$ is a weight function, $\boldsymbol{\Sigma}$ is the unconditional covariance matrix and $\|\cdot\|$ is the L_1 norm. Then, after some calculations, the conditional covariance matrix can be obtained by:

$$\mathbf{H}_t = \mathbf{A}\mathbf{H}_t^f\mathbf{A}' + \mathbf{A}\mathbf{A}'\boldsymbol{\Sigma}\mathbf{B}\mathbf{B}' + \mathbf{B}\mathbf{B}'\boldsymbol{\Sigma}, \tag{16}$$

where $\mathbf{H}_t^f$ is the conditional covariance matrix of the volatility components with volatility evolving over time and the remaining are terms as defined previously[3]. The matrix $\mathbf{G}$ is estimated as:

$$\hat{\mathbf{G}} = \sum_{k=1}^{g}\sum_{\tau=1}^{T}\omega(\mathbf{y}_\tau)\left[\frac{1}{T-k}\sum_{t=k+1}^{T}\left[(\mathbf{y}_t\mathbf{y}_t' - \hat{\boldsymbol{\Sigma}})\, I(\|\mathbf{y}_{t-k}\| \le \|\mathbf{y}_\tau\|)\right]\right]^2. \tag{17}$$

The estimated version of Equation (16) is obtained by replacing the true values with the estimated ones.

2.7. The Robust GPVC Model

Trucíos et al. (2019) show the non-robustness of the GPVC procedure of Li et al. (2016) and propose an alternative procedure to obtain volatility components that is robust to outliers. This procedure is based on a robust estimator of the unconditional covariance matrix, a weighted estimator of $E\left[(\mathbf{y}_t\mathbf{y}_t' - \boldsymbol{\Sigma})\, I(\|\mathbf{y}_{t-k}\| \le \|\mathbf{y}_t\|)\right]$, and robustified filters. The matrix (17) is replaced by a less sensitive matrix, defined as:

$$\hat{\mathbf{G}}^R = \sum_{k=1}^{g}\sum_{\tau=1}^{T}\omega(\mathbf{y}_\tau)\left[\sum_{t=k+1}^{T}\delta^*(d_t^2)\left\{(\mathbf{y}_t\mathbf{y}_t' - \hat{\boldsymbol{\Sigma}}^R) I(\|\mathbf{y}_{t-k}\| \le \|\mathbf{y}_\tau\|)\right\}\right]^2, \tag{18}$$

where d_t^2 is the robust squared Mahalanobis distance given by $d_t^2 = (\mathbf{y}_t - \hat{}^R)'\hat{\boldsymbol{\Sigma}}_t^{-1}(\mathbf{y}_t - \hat{}^R)$ with $\hat{\boldsymbol{\Sigma}}_t = 0.01\rho(\mathbf{y}_{t-1}'\mathbf{y}_{t-1}) + 0.99\hat{\boldsymbol{\Sigma}}_{t-1}$, $\hat{\boldsymbol{\Sigma}}_1 = \hat{\boldsymbol{\Sigma}}^R$ and $\hat{}^R$, $\hat{\boldsymbol{\Sigma}}^R$ being robust estimates of the unconditional mean and covariance matrix. Trucíos et al. (2019) use the minimum covariance determinant (MCD) estimator of Rousseeuw (1984), implemented by the algorithm of Hubert et al. (2012). The robust filters, $\rho(\cdot)$ and $\delta(\cdot)$ are given by $\rho(x_t) = x_t$ if $d_t^2 \le c$, $\rho(x_t) = \hat{\Sigma}^R$ if $d_t^2 > c$; $\delta(x) = 1$ if $x \le c$, $\delta(x) = 1/x$ if $x > c$ and $\delta^*(\cdot) = \delta(\cdot)/||\delta(\cdot)||$, where $\|\cdot\|$ is the L_1 norm. For details, see Trucíos et al. (2019).

To avoid returns corresponding to periods with high volatility being considered as possible outliers, the robust procedure incorporates in the squared Mahalanobis distance a covariance matrix evolving over time, which can be seen as a robust RM1994 method with $\lambda = 0.99$.

Finally, the conditional covariance matrix $\mathbf{H}_t$ is obtained as in Equation (16).

2.8. Linear and Non-Linear Shrinkage

Besides the estimation of the covariance matrix ($\mathbf{H}_t$), in some of the aforementioned models, we have to estimate the unconditional covariance or correlation matrix; for instance, the matrix $\mathbf{C}$ in Equation (7) of the DCC model. Generally, the estimation of the unconditional correlation (covariance) matrix is done using the sample correlation (covariance) matrix. However, this is inefficient in the large dimensional case because we could end up with a number of parameters with the same order of magnitude as the dataset, or even larger (see, for instance, the simulation study in the Appendix of Engle et al. (2017)). In general, comparing the eigenvalues of the true correlation matrix with the eigenvalues of the sample correlation matrix, there is a tendency to underestimate the smaller eigenvalues and overestimate the larger ones. A natural way to reduce this bias is to increase the smaller eigenvalues and decrease the larger sample eigenvalues and then reconstruct the estimate of the

3 Note that when $\boldsymbol{\Sigma} = \mathbf{I}$, $\mathbf{H}_t = \mathbf{A}\mathbf{H}_t^f\mathbf{A}' + \mathbf{B}\mathbf{B}'\boldsymbol{\Sigma} = \mathbf{A}\mathbf{H}_t^f\mathbf{A}' + \boldsymbol{\Sigma}_{\mathbf{ffl}}$ as presented in Li et al. (2016).

correlation matrix. This is the main idea behind the shrinkage method. Engle et al. (2017) analyse the use of three types the shrinkage: linear shrinkage of Ledoit and Wolf (2004b) with shrinkage target given by (a multiple of) the identity matrix; linear shrinkage of Ledoit and Wolf (2004a) with shrinkage target given by the equicorrelation matrix; and the non-linear shrinkage of Ledoit and Wolf (2012) for the estimation of the unconditional correlation matrix in Equation (7). Using simulation, they conclude that the three types of shrinkage have better performance than the use of the sample correlation matrix in the estimation of $\mathbf{H}_t$, and the best performance is obtained from the non-linear shrinkage. They conclude that the application of non-linear shrinkage improves the estimation, and the improvement generally increases for a larger number of assets. In the application, they also apply the non-linear shrinkage to the estimated one-step-ahead conditional covariance matrix, which is not done in the simulation study. In the empirical application, they construct portfolios of global minimum variance with portfolio sizes 100, 500 and 1000 and updated monthly. As in the simulation study, they construct portfolios with H_t modelled by DCC and CCC models and the RiskMetrics 2006 method. However, besides applying the linear and non-linear shrinkage to the target correlation matrix, they also apply the shrinkages to the one-step-ahead prediction of the volatility matrix. The best performance is achieved by the DCC model with the non-linear shrinkage applied only to the estimation of the intercept matrix, followed by the non-linear shrinkage applied both to the intercept matrix and to the one-step-ahead prediction matrix. We use the linear shrinkage towards the equicorrelation matrix, because in Engle et al. (2017) it presented slightly better performance than the shrinkage towards the identity matrix, although the estimator does not belong to the class of rotation-equivariant estimators.

For a light introduction to the main idea behind shrinkage, suppose we want to estimate the covariance matrix $\mathbf{\Sigma}$ and we have an estimate $\hat{\mathbf{C}}$ based on a sample of size T. For instance, $\hat{\mathbf{C}}$ could be the sample covariance matrix and $\mathbf{\Sigma}$, the population matrix (unconditional covariance matrix). This is the case of the estimation of the DCC, where $\mathbf{\Sigma}$ is the intercept matrix. When the ratio N/T, called concentration ratio, becomes large, we have in-sample overfitting due to the excessive number of parameters, introducing a bias in the estimation of the eigenvalues. One way to correct this problem is through the shrinkage method.

For the linear shrinkage towards the equicorrelation matrix, denote by $\hat{c}_{ij}$ the element of the estimate $\hat{C}$. The mean of the estimated correlations is given by:

$$\bar{r} = \frac{2}{(N-1)N} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\hat{c}_{i,j}}{\sqrt{\hat{c}_{i,i}\hat{c}_{j,j}}}, \tag{19}$$

such that for the target matrix F we have $f_{i,i} = \hat{c}_{i,i}$ and $f_{i,j} = \bar{r}\sqrt{\hat{c}_{i,i}\hat{c}_{j,j}}$. The shrinkage estimate is given by:

$$\hat{\mathbf{\Sigma}}_{Shrink} = \delta F + (1-\delta)\hat{C}, \tag{20}$$

where the shrinkage intensity, δ, is such that it minimizes the expected quadratic loss as in Ledoit and Wolf (2004a). For the shrinkage intensity δ, define the quadratic loss function

$$L(\delta) = ||\delta F + (1-\delta)\hat{C} - \Sigma||^2.$$

Ledoit and Wolf (2004a) propose to use the shrinkage intensity, which minimizes the risk function $R(\delta) = E(L(\delta))$. The formulae and the derivation of the estimated shrinkage intensity can be found in the Appendix B of Ledoit and Wolf (2004a).

Regarding the non-linear shrinkage, let $\hat{C}$ having dimension $(N \times N)$, $(\hat{\lambda}_1, \ldots, \hat{\lambda}_N)$, sorted in descending order, be the set of eigenvalues, and $(\hat{\mathbf{u}}_1, \ldots, \hat{\mathbf{u}}_N)$ the corresponding eigenvectors, such that:

$$\hat{C} = \sum_{i=1}^{N} \hat{\lambda}_i \hat{\mathbf{u}}_i \hat{\mathbf{u}}_i'. \tag{21}$$

For an investor holding a portfolio with weights ω, the estimated variance is given by $\omega'\hat{C}\omega$. The non-linear shrinkage of Ledoit and Wolf (2004b) is a transformation from $(\hat{\lambda}_1, \ldots, \hat{\lambda}_N)$ to $\tilde{\lambda} = (\tilde{\lambda}_1, \ldots, \tilde{\lambda}_N)$, such that substituting $\hat{\lambda}_i$ for $\tilde{\lambda}_i$ in Equation (21) gives a consistent estimator of the out-of-sample variance $\omega'\Sigma\omega'$. Denote by $\lambda = (\lambda_1, \ldots, \lambda_N)$ the set of eigenvalues of Σ in descending order. Ledoit and Wolf (2004b) define QuEST functions $(q_1(\lambda), \ldots, q_N(\lambda)$, such that $\tilde{\lambda}$ minimizes the Euclidean distance between the QuEST functions and the sample eigenvalues, i.e., given by:

$$\tilde{\lambda} = \arg \min_{\lambda \in [0,\infty)^N} \sum_{i=1}^{N} [q_i(\lambda) - \hat{\lambda}_i]^2. \tag{22}$$

A definition of the QuEST functions and a rigorous exposition of non-linear shrinkage can be found in Ledoit and Wolf (2012), while a lighter presentation can be found in the Supplementary Material of Engle et al. (2017).

3. Empirical Application

3.1. Data and Methods

In this section, we implement the procedures described in Section 2 and use the predicted one-step-ahead conditional covariance matrix to construct the minimum variance portfolio (MVP) of the stocks used in the composition of the S&P 500 index, traded from 2 January 2000 to 30 November 2017. Because not all stocks of the index were traded during the whole period, we ended up with $N = 174$ stocks.

To evaluate the out-of-sample portfolio performance, we consider a rolling window scheme. The out-of-sample portfolio performance is evaluated in four different periods, namely: pre-crisis period (January 2004 to December 2007, 1008 days), subprime crisis period (January 2008 to June 2009, 378 days), post-crisis period (July 2009 to November 2017, 2218 days), and full period (January 2004 to November 2017, 3503 days). In each window, the one-step-ahead covariance matrix is estimated and the MVP values with and without short-sale constraints are obtained. The weights in the MVP portfolio are rebalanced with both daily and monthly frequencies. In the latter case, we follow Engle et al. (2017), that is, we obtain the portfolio returns daily but update the weights monthly (following the common convention we use 21 consecutive trading days as a month). Monthly updating is common in practice to reduce transaction costs.

The procedures described in Section 2 are combined with the linear and non-linear shrinkage estimator described in Subsection 2.8. The linear and non-linear shrinkage are applied at the beginning and/or at the end of the estimation procedure. A detailed description of each combination of the estimation procedures is given in the Appendix A. In addition, for the sake of comparison, we also implement the naive equal-weighted portfolio. In the line of Engle et al. (2017), Gambacciani and Paolella (2017), Trucíos et al. (2018) among others, we consider the following annualised out-of-sample performance measures. Denote by $R_p = \{r_{p,1}, \ldots, r_{p,k}\}$ the observed out-of-sample returns from a given method where k in the length of the out-of-sample period. The measures considered in this paper: the annualised average portfolio return (AV), standard deviation portfolio return (SD), information ratio (IR), Sortino's ratio (SR) and average turnover (TO) are computed as follows:

AV: equal to $252 \times \bar{R}_p$, where $\bar{R}_p$ is the average of the elements of R_p.
SD: equal to $\sqrt{252} \times S_p$, where S_p is the standard deviation of the elements of R_p.
IR: AV/SD.
SR: AV/$\sqrt{252 \times S^{*2}}$, where S^{*2} is the mean of $r^*_{p,i}, i = 1, \ldots, k$, with $r^*_{p,i} = r^2_{p,i}$ if $r_{p,i}$ less than the minimal acceptable return, which is taken to zero, and zero otherwise.
TO: $k^{-1} \sum_{t=2}^{k} \sum_{j=1}^{N} |\omega_{j,t} - \omega_{j,t-1}|$ where $\omega_{j,t}$ is the portfolio weight at time t for the j-th asset, and k is the number of the out-of-sample portfolio returns.

As pointed out by Kirby and Ostdiek (2012), Santos and Ferreira (2017), Olivares-Nadal and DeMiguel (2018), among others, transaction costs (c) can have an impact on the portfolio's performance. In order to take into account those costs, we also compute the portfolio returns net of transaction cost. For a given c, the portfolio return net of transaction costs at time t is given by $r_{p,t}^{net} = (1 - c \times turnover_t)(1 + r_{p,t}) - 1$ and then the annualised average portfolio return net of transaction costs is $\text{AV}^{net} = 252 \times \bar{R}_p^{net}$ where $\bar{R}_p^{net}$ is the average of the portfolio return net of transaction costs $r_{p,1}^{net}, \ldots, r_{p,k}^{net}$. We consider $c = 20bp$ (intermediate) and $c = 50bp$ (high level) transaction costs where a basis point (bp) is a unit of measure commonly used in finance and is equivalent to 0.01%. The annualised average portfolio return net of transation costs considering $c = 20bp$ and $c = 50bp$ are denoted by AV_{20bp}^{net} and AV_{50bp}^{net}, respectively.

3.2. Results

Tables 1–8 report annualised out-of-sample performance measures for MVP with performance for the pre-crisis, crisis, post-crisis and full periods. Tables 1–4 report the results for daily rebalanced portfolios whereas Tables 5–8 report the results for monthly rebalanced portfolios. We also have results for MVP with no short-sale constraints. However, in this paper we focus on the results for MVP with short-sale constraints and give a short summary of the main findings for the case without short-sale constraints. A detailed analysis of the case without short-sale constraints is given in the Supplementary Material.

In Tables 1–8 we report (in parentheses) the rank of the methods according to the SD criterion in the second column. Moreover, for each criterion, the best five methods are highlighted in shadowed cells. The equal-weighted portfolio strategy is represented by $1/N$.

Taking into account the fact that portfolios are chosen in order to have the minimum variance, the analysis is first done according to the SD criterion. For portfolios rebalanced daily or monthly, the largest SD is reported by the equal-weight portfolio strategy. For portfolios rebalanced daily (Tables 1–4), the five smallest SDs are obtained by the DCC based-methods, except in the crisis period, in which case the five smallest SDs are spread among the DCC, OGARCH and GPVC based-methods. In the crisis-period, the smallest SD is obtained by the GPVC procedure with the non-linear shrinkage applied to the one-step-ahead conditional covariance matrix. For portfolios rebalanced monthly (Tables 5–8), the smallest SDs are obtained by the RM2006-LS[4], NLS-DCC, NLS-GPVC and RM2006-LS procedures for the full, pre-crisis, crisis and post-crisis periods, respectively.

The best performance in terms of the AV criterion differs depending on the period and rebalance strategy. For instance, for daily rebalancing the best performance in the full period is achieved by the RPVC followed by the RPVC with non-linear shrinkage applied to the one-step-ahead conditional covariance matrix. However, for the pre-crisis, crises and post-crisis periods, the best performance is achieved by the OGARCH with non-linear shrinkage applied to the unconditional covariance matrix (NLS-OGARCH), RPVC with linear shrinkage applied to the one-step-ahead conditional covariance matrix (RPVC-LS) and RiskMetrics method with linear shrinkage applied to the one-step-ahead conditional covariance matrix (RM1994-LS), respectively. For monthly rebalancing, the best performances in the full, pre-crisis, crisis and post-crisis periods are achieved by the RPVC, OGARCH-NLS, GPVC-LS and equal-weight portfolio strategy, respectively.

In terms of average turnover, the five smallest average turnovers are in the OGARCH and GPVC groups, with the best performance being achieved by the OGARCH with non-linear shrinkage applied to the one-step-ahead conditional covariance matrix in almost all cases. The only two exceptions are observed in the crisis period, in which case the best performance is achieved by the GPVC procedure with non-linear shrinkage applied to the one-step-ahead conditional covariance matrix.

[4] The acronyms are described in the Appendix A.

Additionally, note that regardless of whether portfolio is rebalanced daily or monthly, the average turnover reported by all dimension reduction techniques is smaller than reported by the non-dimension reduction procedures.

Table 1. Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns. AV^{net}_{20bp} and AV^{net}_{50bp} stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period January 2004 to November 2017. The shaded cells denote the top five for each criterion. Weights are rebalanced on a daily basis considering short-selling constraints.

	AV	**SD**	**IR**	**SR**	**TO**	$\mathbf{AV^{net}_{20bp}}$	$\mathbf{AV^{net}_{50bp}}$
1/N	8.302	20.058 (47)	0.414	0.570	-	-	-
CCC	7.706	11.839 (12)	0.651	0.890	0.297	7.509	7.279
CCC LS	7.004	11.881 (14)	0.590	0.807	0.307	6.815	6.578
CCC NLS	7.876	11.932 (17)	0.660	0.905	0.277	7.685	7.470
LS CCC	7.506	11.816 (11)	0.635	0.868	0.302	7.311	7.078
NLS CCC	7.345	11.809 (10)	0.622	0.848	0.298	7.153	6.923
LS CCC LS	6.628	11.918 (16)	0.556	0.759	0.305	6.439	6.205
NLS CCC NLS	7.522	11.910 (15)	0.632	0.865	0.303	7.327	7.091
DCC	7.737	11.613 (2)	0.666	0.908	0.308	7.532	7.296
DCC LS	6.941	11.689 (5)	0.594	0.810	0.314	6.749	6.508
DCC NLS	7.711	11.695 (6)	0.659	0.905	0.285	7.513	7.292
LS DCC	7.707	11.613 (1)	0.664	0.904	0.308	7.502	7.266
NLS DCC	7.629	11.616 (3)	0.657	0.894	0.307	7.424	7.188
LS DCC LS	6.907	11.688 (4)	0.591	0.806	0.314	6.715	6.474
NLS DCC NLS	7.645	11.699 (7)	0.653	0.896	0.283	7.447	7.227
RM2006	8.649	11.809 (9)	0.732	0.995	0.271	8.446	8.234
RM2006 LS	8.746	11.724 (8)	0.746	1.017	0.282	8.564	8.343
RM2006 NLS	8.734	11.865 (13)	0.736	1.011	0.268	8.537	8.327
RM1994	8.502	12.220 (22)	0.696	0.947	0.283	8.289	8.069
RM1994 LS	8.391	12.012 (18)	0.699	0.953	0.277	8.196	7.979
RM1994 NLS	8.763	12.151 (19)	0.721	0.990	0.225	8.581	8.405
DECO	5.980	12.258 (25)	0.488	0.660	0.297	5.797	5.568
DECO NLS	6.103	12.485 (41)	0.489	0.669	0.360	5.884	5.604
LS DECO	5.980	12.257 (24)	0.488	0.660	0.297	5.797	5.568
NLS DECO	5.981	12.257 (23)	0.488	0.660	0.297	5.798	5.569
NLS DECO NLS	6.103	12.485 (42)	0.489	0.669	0.360	5.884	5.604
OGARCH	8.363	12.341 (27)	0.678	0.936	0.095	8.271	8.196
OGARCH LS	7.052	12.544 (43)	0.562	0.773	0.103	6.974	6.893
OGARCH NLS	8.126	12.154 (20)	0.669	0.928	0.072	8.052	7.996
LS OGARCH	7.951	12.477 (39)	0.637	0.877	0.095	7.860	7.786
NLS OGARCH	8.365	12.341 (27)	0.678	0.936	0.095	8.273	8.198
LS OGARCH LS	6.880	12.710 (44)	0.541	0.743	0.101	6.802	6.723
NLS OGARCH NLS	8.126	12.154 (20)	0.669	0.928	0.072	8.051	7.996
GPVC	7.825	12.467 (38)	0.628	0.861	0.132	7.700	7.598
GPVC LS	7.438	12.274 (26)	0.606	0.834	0.106	7.341	7.259
GPVC NLS	6.727	12.369 (31)	0.544	0.749	0.113	6.621	6.533
LS GPVC	7.994	12.452 (36)	0.642	0.891	0.117	7.872	7.781
NLS GPVC	7.672	12.433 (33)	0.617	0.845	0.130	7.547	7.447
LS GPVC LS	7.470	12.429 (32)	0.601	0.826	0.161	7.359	7.238
NLS GPVC NLS	6.725	12.365 (30)	0.544	0.749	0.113	6.619	6.533
RPVC	9.657	12.785 (45)	0.755	1.047	0.222	9.479	9.310
RPCV LS	7.989	12.439 (34)	0.642	0.889	0.180	7.861	7.724
RPVC NLS	9.186	12.485 (40)	0.736	1.026	0.184	9.035	8.893
LS RPVC	8.543	12.347 (29)	0.692	0.953	0.201	8.387	8.235
NLS RPVC	8.064	13.142 (46)	0.614	0.850	0.191	7.904	7.755
LS RPCV LS	7.493	12.439 (35)	0.602	0.828	0.167	7.378	7.252
NLS RPVC NLS	7.658	12.460 (37)	0.615	0.850	0.172	7.509	7.376

Table 2. Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns. AV^{net}_{20bp} and AV^{net}_{50bp} stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period January 2004 to December 2007. The shaded cells denote the top five for each criterion. Weights are rebalanced on a daily basis considering short-selling constraints.

	AV	**SD**	**IR**	**SR**	**TO**	$\mathbf{AV^{net}_{20bp}}$	$\mathbf{AV^{net}_{50bp}}$
1/N	12.732	12.755 (47)	0.998	1.418	-	-	-
CCC	11.425	8.381 (6)	1.363	1.963	0.256	11.137	10.934
CCC LS	9.818	8.495 (14)	1.156	1.655	0.264	9.569	9.362
CCC NLS	11.157	8.404 (11)	1.328	1.907	0.247	10.863	10.668
LS CCC	11.305	8.394 (9)	1.347	1.940	0.258	11.030	10.826
NLS CCC	11.461	8.399 (10)	1.365	1.966	0.251	11.195	10.997
LS CCC LS	9.632	8.628 (17)	1.116	1.596	0.258	9.386	9.183
NLS CCC NLS	11.172	8.426 (12)	1.326	1.910	0.263	10.901	10.692
DCC	11.144	8.203 (3)	1.359	1.947	0.263	10.843	10.636
DCC LS	9.450	8.394 (8)	1.126	1.605	0.268	9.201	8.992
DCC NLS	10.919	8.234 (5)	1.326	1.898	0.253	10.609	10.410
LS DCC	11.103	8.199 (2)	1.354	1.941	0.263	10.802	10.596
NLS DCC	11.035	8.196 (1)	1.346	1.929	0.262	10.733	10.527
LS DCC LS	9.423	8.391 (7)	1.123	1.601	0.268	9.174	8.965
NLS DCC NLS	10.829	8.226 (4)	1.316	1.884	0.252	10.519	10.321
RM2006	11.983	8.553 (15)	1.401	2.045	0.258	11.630	11.426
RM2006 LS	10.988	8.435 (13)	1.303	1.887	0.268	10.728	10.516
RM2006 NLS	9.852	8.686 (19)	1.134	1.619	0.259	9.520	9.318
RM1994	9.496	9.148 (29)	1.038	1.503	0.282	9.121	8.902
RM1994 LS	8.498	8.866 (23)	0.959	1.374	0.275	8.182	7.967
RM1994 NLS	10.080	9.112 (28)	1.106	1.584	0.220	9.742	9.571
DECO	9.282	9.062 (25)	1.024	1.457	0.253	9.040	8.840
DECO NLS	8.998	9.197 (32)	0.978	1.388	0.302	8.725	8.487
LS DECO	9.280	9.063 (26)	1.024	1.456	0.253	9.039	8.838
NLS DECO	9.271	9.064 (27)	1.023	1.455	0.254	9.030	8.829
NLS DECO NLS	8.998	9.197 (33)	0.978	1.388	0.302	8.725	8.487
OGARCH	13.356	9.188 (31)	1.454	2.097	0.083	13.165	13.100
OGARCH LS	11.565	10.105 (45)	1.144	1.602	0.088	11.435	11.367
OGARCH NLS	12.805	9.203 (34)	1.391	1.998	0.071	12.638	12.582
LS OGARCH	13.068	9.257 (36)	1.412	2.030	0.081	12.885	12.821
NLS OGARCH	13.362	9.188 (30)	1.454	2.098	0.083	13.172	13.106
LS OGARCH LS	11.305	10.326 (46)	1.095	1.528	0.082	11.175	11.110
NLS OGARCH NLS	12.804	9.203 (34)	1.391	1.997	0.071	12.637	12.582
GPVC	11.497	9.268 (37)	1.241	1.757	0.109	11.246	11.163
GPVC LS	11.024	9.282 (39)	1.188	1.680	0.082	10.835	10.772
GPVC NLS	11.210	9.320 (43)	1.203	1.690	0.099	10.993	10.918
LS GPVC	12.213	9.294 (41)	1.314	1.868	0.094	11.953	11.881
NLS GPVC	11.274	9.348 (44)	1.206	1.703	0.108	11.020	10.938
LS GPVC LS	10.325	9.288 (40)	1.112	1.559	0.129	10.153	10.052
NLS GPVC NLS	11.165	9.318 (42)	1.198	1.683	0.097	10.949	10.876
RPVC	12.966	8.680 (18)	1.494	2.169	0.193	12.642	12.492
RPCV LS	10.423	9.000 (24)	1.158	1.646	0.152	10.218	10.100
RPVC NLS	12.233	8.697 (20)	1.407	2.018	0.171	11.951	11.818
LS RPVC	11.635	8.577 (16)	1.357	1.944	0.175	11.354	11.218
NLS RPVC	10.878	8.829 (22)	1.232	1.760	0.171	10.579	10.447
LS RPCV LS	10.304	9.271 (38)	1.111	1.558	0.139	10.125	10.016
NLS RPVC NLS	10.628	8.760 (21)	1.213	1.723	0.158	10.336	10.215

Table 3. Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns. AV^{net}_{20bp} and AV^{net}_{50bp} stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period January 2008 to June 2009. The shaded cells denote the top five for each criterion. Weights are rebalanced on a daily basis considering short-selling constraints.

	AV	SD	IR	SR	TO	$\mathbf{AV^{net}_{20bp}}$	$\mathbf{AV^{net}_{50bp}}$
1/N	−30.668	43.046 (47)	−0.713	−0.960	-	-	-
CCC	−25.407	22.009 (20)	−1.154	−1.464	0.362	−25.564	−25.799
CCC LS	−25.522	22.003 (19)	−1.160	−1.471	0.365	−25.680	−25.917
CCC NLS	−23.682	22.613 (27)	−1.047	−1.344	0.300	−23.820	−24.026
LS CCC	−26.288	21.934 (13)	−1.199	−1.516	0.369	−26.448	−26.686
NLS CCC	−27.144	21.965 (16)	−1.236	−1.558	0.365	−27.301	−27.537
LS CCC LS	−27.052	21.967 (17)	−1.232	−1.553	0.368	−27.211	−27.449
NLS CCC NLS	−25.372	22.346 (25)	−1.135	−1.446	0.326	−25.521	−25.743
DCC	−26.520	21.580 (5)	−1.229	−1.554	0.389	−26.683	−26.928
DCC LS	−26.702	21.596 (7)	−1.236	−1.563	0.391	−26.866	−27.112
DCC NLS	−24.636	21.926 (12)	−1.124	−1.446	0.312	−24.777	−24.989
LS DCC	−26.639	21.582 (6)	−1.234	−1.561	0.390	−26.802	−27.047
NLS DCC	−27.020	21.596 (7)	−1.251	−1.581	0.392	−27.184	−27.429
LS DCC LS	−26.833	21.599 (9)	−1.242	−1.570	0.392	−26.997	−27.243
NLS DCC NLS	−24.899	21.952 (14)	−1.134	−1.460	0.311	−25.039	−25.249
RM2006	−22.728	21.862 (11)	−1.040	−1.326	0.281	−22.858	−23.054
RM2006 LS	−22.912	21.815 (10)	−1.050	−1.338	0.279	−23.041	−23.235
RM2006 NLS	−21.267	21.958 (15)	−0.969	−1.264	0.216	−21.372	−21.529
RM1994	−20.793	22.108 (22)	−0.941	−1.205	0.260	−20.914	−21.096
RM1994 LS	−21.234	22.053 (21)	−0.963	−1.232	0.259	−21.355	−21.537
RM1994 NLS	−20.974	22.161 (23)	−0.946	−1.236	0.178	−21.060	−21.188
DECO	−31.859	22.706 (33)	−1.403	−1.742	0.408	−32.030	−32.288
DECO NLS	−29.187	22.618 (28)	−1.291	−1.633	0.386	−29.358	−29.615
LS DECO	−31.854	22.706 (32)	−1.403	−1.742	0.408	−32.026	−32.284
NLS DECO	−31.829	22.702 (31)	−1.402	−1.741	0.408	−32.001	−32.258
NLS DECO NLS	−29.188	22.618 (29)	−1.291	−1.633	0.386	−29.359	−29.615
OGARCH	−21.671	23.390 (36)	−0.927	−1.218	0.107	−21.722	−21.799
OGARCH LS	−21.745	23.360 (35)	−0.931	−1.223	0.108	−21.796	−21.873
OGARCH NLS	−20.118	21.541 (3)	−0.934	−1.223	0.071	−20.153	−20.205
LS OGARCH	−23.677	24.009 (45)	−0.986	−1.291	0.109	−23.728	−23.804
NLS OGARCH	−21.671	23.390 (36)	−0.927	−1.218	0.107	−21.722	−21.799
LS OGARCH LS	−23.571	23.957 (41)	−0.984	−1.288	0.109	−23.622	−23.699
NLS OGARCH NLS	−20.118	21.541 (3)	−0.934	−1.223	0.071	−20.153	−20.205
GPVC	−19.789	22.287 (24)	−0.888	−1.151	0.105	−19.831	−19.894
GPVC LS	−16.841	22.700 (30)	−0.742	−0.973	0.113	−16.890	−16.964
GPVC NLS	−23.692	21.444 (1)	−1.105	−1.434	0.050	−23.711	−23.740
LS GPVC	−18.380	22.823 (34)	−0.805	−1.079	0.112	−18.429	−18.503
NLS GPVC	−20.574	21.983 (18)	−0.936	−1.207	0.102	−20.614	−20.674
LS GPVC LS	−21.137	23.982 (43)	−0.881	−1.144	0.193	−21.208	−21.315
NLS GPVC NLS	−23.716	21.451 (2)	−1.106	−1.435	0.050	−23.735	−23.764
RPVC	−17.369	23.870 (40)	−0.728	−0.962	0.188	−17.446	−17.561
RPCV LS	−15.911	23.839 (39)	−0.667	−0.888	0.189	−15.990	−16.109
RPVC NLS	−22.229	22.432 (26)	−0.991	−1.296	0.114	−22.277	−22.350
LS RPVC	−21.004	23.672 (38)	−0.887	−1.153	0.195	−21.076	−21.183
NLS RPVC	−25.119	27.169 (46)	−0.925	−1.231	0.156	−25.192	−25.302
LS RPCV LS	−21.164	23.982 (44)	−0.883	−1.145	0.193	−21.235	−21.342
NLS RPVC NLS	−25.492	23.964 (42)	−1.064	−1.389	0.115	−25.543	−25.620

Table 4. Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns. AV^{net}_{20bp} and AV^{net}_{50bp} stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period July 2009 to November 2017. The shaded cells denote the top five for each criterion. Weights are rebalanced on a daily basis considering short-selling constraints.

	AV	SD	IR	SR	TO	AV^{net}_{20bp}	AV^{net}_{50bp}
1/N	13.130	16.057 (47)	0.818	1.148	-	-	-
CCC	11.830	10.561 (15)	1.120	1.606	0.306	11.669	11.427
CCC LS	11.455	10.599 (18)	1.081	1.554	0.318	11.288	11.037
CCC NLS	11.932	10.502 (10)	1.136	1.628	0.288	11.781	11.554
LS CCC	11.713	10.540 (13)	1.111	1.595	0.310	11.550	11.304
NLS CCC	11.525	10.512 (11)	1.096	1.571	0.308	11.362	11.119
LS CCC LS	11.193	10.629 (19)	1.053	1.514	0.315	11.027	10.778
NLS CCC NLS	11.640	10.552 (14)	1.103	1.583	0.317	11.473	11.222
DCC	12.213	10.366 (1)	1.178	1.681	0.315	12.047	11.797
DCC LS	11.736	10.429 (8)	1.125	1.612	0.322	11.566	11.311
DCC NLS	11.942	10.383 (5)	1.150	1.644	0.295	11.786	11.553
LS DCC	12.204	10.366 (1)	1.177	1.679	0.314	12.038	11.788
NLS DCC	12.175	10.367 (3)	1.174	1.674	0.314	12.009	11.761
LS DCC LS	11.715	10.427 (7)	1.124	1.609	0.322	11.545	11.290
NLS DCC NLS	11.922	10.383 (4)	1.148	1.641	0.293	11.768	11.536
RM2006	12.648	10.498 (9)	1.205	1.686	0.275	12.502	12.283
RM2006 LS	13.314	10.403 (6)	1.280	1.812	0.289	13.160	12.930
RM2006 NLS	13.542	10.518 (12)	1.288	1.820	0.281	13.393	13.169
RM1994	13.243	10.941 (35)	1.210	1.691	0.287	13.091	12.863
RM1994 LS	13.613	10.686 (25)	1.274	1.799	0.281	13.463	13.239
RM1994 NLS	13.430	10.808 (32)	1.243	1.747	0.235	13.305	13.117
DECO	11.144	10.806 (29)	1.031	1.478	0.298	10.986	10.749
DECO NLS	11.007	11.214 (39)	0.982	1.410	0.383	10.805	10.501
LS DECO	11.145	10.806 (31)	1.031	1.478	0.298	10.987	10.750
NLS DECO	11.145	10.806 (29)	1.031	1.478	0.298	10.987	10.750
NLS DECO NLS	11.007	11.214 (39)	0.982	1.410	0.383	10.805	10.501
OGARCH	11.333	10.671 (22)	1.062	1.508	0.098	11.280	11.201
OGARCH LS	10.030	10.684 (24)	0.939	1.334	0.109	9.972	9.885
OGARCH NLS	10.927	11.000 (36)	0.993	1.422	0.072	10.889	10.833
LS OGARCH	11.145	10.658 (20)	1.046	1.485	0.099	11.092	11.012
NLS OGARCH	11.333	10.671 (22)	1.062	1.508	0.098	11.280	11.201
LS OGARCH LS	10.194	10.669 (21)	0.956	1.360	0.108	10.136	10.050
NLS OGARCH NLS	10.926	11.000 (37)	0.993	1.421	0.072	10.889	10.833
GPVC	10.992	11.289 (41)	0.974	1.377	0.148	10.913	10.795
GPVC LS	10.052	10.781 (27)	0.932	1.324	0.116	9.991	9.898
GPVC NLS	10.008	11.374 (44)	0.880	1.251	0.132	9.939	9.835
LS GPVC	10.681	11.061 (38)	0.966	1.366	0.128	10.612	10.508
NLS GPVC	10.985	11.300 (42)	0.972	1.375	0.146	10.907	10.790
LS GPVC LS	11.203	10.569 (16)	1.060	1.532	0.170	11.114	10.981
NLS GPVC NLS	10.030	11.364 (43)	0.883	1.256	0.131	9.962	9.858
RPVC	12.892	11.524 (46)	1.119	1.592	0.241	12.766	12.578
RPCV LS	11.084	10.772 (26)	1.029	1.466	0.192	10.985	10.836
RPVC NLS	13.327	11.476 (45)	1.161	1.682	0.202	13.221	13.062
LS RPVC	12.331	10.816 (33)	1.140	1.636	0.214	12.219	12.051
NLS RPVC	12.630	10.801 (28)	1.169	1.677	0.207	12.521	12.358
LS RPCV LS	11.256	10.596 (17)	1.062	1.535	0.177	11.164	11.026
NLS RPVC NLS	12.145	10.837 (34)	1.121	1.619	0.189	12.047	11.898

Table 5. Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns. AV^{net}_{20bp} and AV^{net}_{50bp} stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period January 2004 to November 2017. The shaded cells denote the top five for each criterion. Weights are rebalanced on a monthly basis considering short-selling constraints.

	AV	SD	IR	SR	TO	AV^{net}_{20bp}	AV^{net}_{50bp}
1/N	8.302	20.058 (47)	0.414	0.570	-	-	-
CCC	7.946	12.262 (10)	0.648	0.902	0.319	7.938	7.925
CCC LS	6.832	12.263 (11)	0.557	0.775	0.329	6.823	6.810
CCC NLS	7.725	12.388 (16)	0.624	0.867	0.296	7.717	7.704
LS CCC	7.731	12.261 (9)	0.631	0.878	0.323	7.723	7.709
NLS CCC	7.588	12.278 (14)	0.618	0.859	0.319	7.579	7.566
LS CCC LS	6.471	12.359 (15)	0.524	0.728	0.325	6.462	6.449
NLS CCC NLS	7.758	12.439 (20)	0.624	0.869	0.321	7.749	7.736
DCC	7.425	12.182 (3)	0.610	0.845	0.325	7.416	7.403
DCC LS	6.567	12.200 (6)	0.538	0.747	0.334	6.558	6.544
DCC NLS	6.901	12.247 (8)	0.563	0.780	0.302	6.892	6.879
LS DCC	7.386	12.184 (4)	0.606	0.840	0.325	7.377	7.364
NLS DCC	7.296	12.193 (5)	0.598	0.829	0.325	7.287	7.274
LS DCC LS	6.518	12.203 (7)	0.534	0.741	0.334	6.509	6.495
NLS DCC NLS	6.781	12.266 (12)	0.553	0.764	0.300	6.772	6.760
RM2006	7.350	12.012 (2)	0.612	0.843	0.287	7.342	7.329
RM2006 LS	7.442	11.870 (1)	0.627	0.867	0.294	7.434	7.421
RM2006 NLS	7.101	12.274 (13)	0.579	0.798	0.296	7.093	7.081
RM1994	7.777	12.644 (29)	0.615	0.848	0.296	7.769	7.756
RM1994 LS	7.157	12.391 (17)	0.578	0.796	0.292	7.149	7.136
RM1994 NLS	7.906	12.606 (27)	0.627	0.865	0.254	7.899	7.888
DECO	5.631	12.899 (43)	0.437	0.608	0.317	5.622	5.609
DECO NLS	5.641	13.162 (44)	0.429	0.599	0.386	5.630	5.614
LS DECO	5.631	12.899 (42)	0.437	0.608	0.317	5.622	5.609
NLS DECO	5.631	12.899 (41)	0.437	0.608	0.317	5.622	5.609
NLS DECO NLS	5.640	13.162 (45)	0.429	0.599	0.386	5.630	5.614
OGARCH	7.819	12.556 (24)	0.623	0.859	0.101	7.816	7.812
OGARCH LS	6.848	12.687 (32)	0.540	0.744	0.113	6.845	6.840
OGARCH NLS	7.985	12.451 (22)	0.641	0.891	0.078	7.984	7.981
LS OGARCH	7.581	12.716 (37)	0.596	0.821	0.103	7.579	7.575
NLS OGARCH	7.821	12.555 (23)	0.623	0.859	0.101	7.818	7.814
LS OGARCH LS	7.029	12.893 (40)	0.545	0.751	0.111	7.026	7.021
NLS OGARCH NLS	7.993	12.451 (21)	0.642	0.891	0.078	7.991	7.988
GPVC	7.282	12.707 (34)	0.573	0.789	0.155	7.277	7.271
GPVC LS	7.225	12.435 (19)	0.581	0.801	0.120	7.222	7.218
GPVC NLS	6.560	12.672 (31)	0.518	0.712	0.132	6.557	6.552
LS GPVC	7.200	12.713 (36)	0.566	0.783	0.138	7.196	7.190
NLS GPVC	7.223	12.697 (33)	0.569	0.782	0.153	7.219	7.212
LS GPVC LS	6.521	12.568 (25)	0.519	0.718	0.172	6.516	6.509
NLS GPVC NLS	6.568	12.665 (30)	0.519	0.713	0.130	6.565	6.559
RPVC	8.453	12.712 (35)	0.665	0.920	0.248	8.446	8.436
RPCV LS	7.355	12.415 (18)	0.592	0.822	0.193	7.350	7.342
RPVC NLS	8.011	12.816 (39)	0.625	0.863	0.201	8.005	7.997
LS RPVC	7.000	12.615 (28)	0.555	0.765	0.227	6.994	6.985
NLS RPVC	6.488	13.243 (46)	0.490	0.676	0.203	6.482	6.474
LS RPCV LS	6.535	12.588 (26)	0.519	0.718	0.180	6.530	6.523
NLS RPVC NLS	6.874	12.741 (38)	0.540	0.743	0.182	6.869	6.862

Table 6. Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns. AV^{net}_{20bp} and AV^{net}_{50bp} stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period January 2004 to December 2007. The shaded cells denote the top five for each criterion. Weights are rebalanced on a monthly basis considering short-selling constraints.

	AV	SD	IR	SR	TO	AV^{net}_{20bp}	AV^{net}_{50bp}
1/N	12.732	12.755 (47)	0.998	1.418	-	-	-
CCC	10.636	8.697 (7)	1.223	1.750	0.265	10.629	10.618
CCC LS	7.605	8.868 (19)	0.858	1.208	0.284	7.597	7.585
CCC NLS	10.356	8.732 (8)	1.186	1.694	0.254	10.349	10.338
LS CCC	10.158	8.738 (9)	1.163	1.659	0.269	10.150	10.139
NLS CCC	10.186	8.758 (11)	1.163	1.659	0.263	10.179	10.168
LS CCC LS	7.319	9.045 (23)	0.809	1.137	0.281	7.312	7.300
NLS CCC NLS	9.802	8.809 (16)	1.113	1.581	0.277	9.795	9.784
DCC	10.939	8.612 (3)	1.270	1.825	0.271	10.932	10.920
DCC LS	7.691	8.796 (15)	0.874	1.233	0.286	7.683	7.671
DCC NLS	10.763	8.661 (6)	1.243	1.782	0.260	10.756	10.745
LS DCC	10.923	8.608 (2)	1.269	1.823	0.271	10.915	10.904
NLS DCC	10.889	8.599 (1)	1.266	1.819	0.269	10.882	10.871
LS DCC LS	7.672	8.795 (14)	0.872	1.230	0.284	7.664	7.653
NLS DCC NLS	10.725	8.649 (5)	1.240	1.778	0.258	10.718	10.707
RM2006	10.378	8.765 (13)	1.184	1.706	0.292	10.369	10.357
RM2006 LS	9.295	8.629 (4)	1.077	1.540	0.300	9.287	9.275
RM2006 NLS	9.578	8.884 (20)	1.078	1.527	0.313	9.569	9.556
RM1994	8.112	9.545 (37)	0.850	1.209	0.323	8.103	8.089
RM1994 LS	6.813	9.279 (24)	0.734	1.033	0.317	6.804	6.791
RM1994 NLS	9.912	9.282 (25)	1.068	1.520	0.265	9.904	9.892
DECO	6.883	9.577 (39)	0.719	1.009	0.277	6.875	6.864
DECO NLS	6.257	9.784 (43)	0.640	0.887	0.340	6.247	6.233
LS DECO	6.882	9.577 (39)	0.719	1.008	0.277	6.875	6.863
NLS DECO	6.873	9.577 (41)	0.718	1.007	0.277	6.865	6.854
NLS DECO NLS	6.257	9.784 (44)	0.640	0.887	0.340	6.247	6.233
OGARCH	12.682	9.305 (26)	1.363	1.958	0.088	12.680	12.676
OGARCH LS	11.229	10.166 (45)	1.105	1.556	0.097	11.226	11.222
OGARCH NLS	12.878	9.376 (29)	1.374	1.971	0.063	12.877	12.874
LS OGARCH	12.588	9.346 (28)	1.347	1.928	0.088	12.586	12.582
NLS OGARCH	12.682	9.305 (26)	1.363	1.958	0.088	12.680	12.676
LS OGARCH LS	11.414	10.359 (46)	1.102	1.548	0.090	11.411	11.408
NLS OGARCH NLS	12.878	9.376 (29)	1.374	1.971	0.063	12.877	12.874
GPVC	11.014	9.504 (36)	1.159	1.636	0.145	11.010	11.004
GPVC LS	11.064	9.438 (31)	1.172	1.657	0.105	11.061	11.057
GPVC NLS	10.637	9.478 (33)	1.122	1.569	0.134	10.634	10.628
LS GPVC	11.235	9.595 (42)	1.171	1.652	0.120	11.232	11.226
NLS GPVC	10.939	9.576 (38)	1.142	1.611	0.145	10.935	10.929
LS GPVC LS	9.183	9.503 (35)	0.966	1.345	0.139	9.179	9.174
NLS GPVC NLS	10.656	9.473 (32)	1.125	1.572	0.132	10.652	10.647
RPVC	11.558	8.741 (10)	1.322	1.896	0.216	11.552	11.544
RPCV LS	10.172	9.038 (22)	1.126	1.594	0.174	10.168	10.161
RPVC NLS	11.023	8.761 (12)	1.258	1.791	0.193	11.018	11.010
LS RPVC	9.859	8.845 (18)	1.115	1.566	0.202	9.854	9.846
NLS RPVC	9.802	8.925 (21)	1.098	1.558	0.193	9.797	9.789
LS RPCV LS	9.188	9.490 (34)	0.968	1.346	0.151	9.184	9.178
NLS RPVC NLS	9.995	8.828 (17)	1.132	1.600	0.183	9.990	9.982

Table 7. Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns. AV^{net}_{20bp} and AV^{net}_{50bp} stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period January 2008 to June 2009. The shaded cells denote the top five for each criterion. Weights are rebalanced on a monthly basis considering short-selling constraints.

	AV	SD	IR	SR	TO	$\mathbf{AV^{net}_{20bp}}$	$\mathbf{AV^{net}_{50bp}}$
1/N	−30.668	43.046 (47)	−0.713	−0.960	-	-	-
CCC	−25.344	22.796 (13)	−1.112	−1.460	0.381	−25.355	−25.371
CCC LS	−25.390	22.796 (14)	−1.114	−1.462	0.383	−25.401	−25.418
CCC NLS	−23.540	23.614 (33)	−0.997	−1.318	0.318	−23.550	−23.566
LS CCC	−25.295	22.748 (12)	−1.112	−1.463	0.385	−25.306	−25.323
NLS CCC	−26.760	22.916 (21)	−1.168	−1.532	0.373	−26.771	−26.787
LS CCC LS	−26.818	22.925 (22)	−1.170	−1.535	0.375	−26.828	−26.844
NLS CCC NLS	−25.222	23.536 (26)	−1.072	−1.414	0.335	−25.233	−25.248
DCC	−26.146	22.840 (16)	−1.145	−1.508	0.419	−26.158	−26.175
DCC LS	−26.248	22.850 (17)	−1.149	−1.513	0.419	−26.259	−26.276
DCC NLS	−23.982	23.354 (24)	−1.027	−1.359	0.333	−23.993	−24.008
LS DCC	−26.384	22.858 (18)	−1.154	−1.520	0.419	−26.395	−26.412
NLS DCC	−27.056	22.905 (20)	−1.181	−1.554	0.419	−27.067	−27.084
LS DCC LS	−26.495	22.865 (19)	−1.159	−1.526	0.421	−26.506	−26.523
NLS DCC NLS	−24.849	23.458 (25)	−1.059	−1.401	0.331	−24.859	−24.875
RM2006	−22.356	22.084 (4)	−1.012	−1.327	0.356	−22.367	−22.383
RM2006 LS	−23.045	22.006 (2)	−1.047	−1.370	0.356	−23.055	−23.071
RM2006 NLS	−21.109	23.116 (23)	−0.913	−1.206	0.274	−21.116	−21.126
RM1994	−22.685	22.716 (11)	−0.999	−1.307	0.337	−22.695	−22.711
RM1994 LS	−23.388	22.619 (10)	−1.034	−1.350	0.335	−23.398	−23.413
RM1994 NLS	−21.739	23.572 (28)	−0.922	−1.215	0.235	−21.745	−21.755
DECO	−28.184	24.101 (42)	−1.169	−1.550	0.404	−28.197	−28.215
DECO NLS	−27.588	23.858 (35)	−1.156	−1.533	0.367	−27.599	−27.617
LS DECO	−28.182	24.100 (41)	−1.169	−1.550	0.404	−28.195	−28.213
NLS DECO	−28.166	24.098 (40)	−1.169	−1.549	0.404	−28.178	−28.197
NLS DECO NLS	−27.591	23.859 (36)	−1.156	−1.533	0.367	−27.602	−27.620
OGARCH	−20.677	23.592 (30)	−0.877	−1.145	0.124	−20.680	−20.683
OGARCH LS	−20.855	23.577 (29)	−0.885	−1.155	0.126	−20.857	−20.860
OGARCH NLS	−19.608	22.343 (7)	−0.878	−1.156	0.063	−19.610	−19.613
LS OGARCH	−20.516	24.433 (44)	−0.840	−1.098	0.130	−20.518	−20.522
NLS OGARCH	−20.677	23.592 (30)	−0.877	−1.145	0.124	−20.680	−20.683
LS OGARCH LS	−20.564	24.390 (43)	−0.843	−1.103	0.132	−20.567	−20.570
NLS OGARCH NLS	−19.608	22.343 (7)	−0.878	−1.156	0.061	−19.610	−19.613
GPVC	−14.454	22.017 (3)	−0.657	−0.868	0.138	−14.457	−14.462
GPVC LS	−14.100	22.418 (9)	−0.629	−0.831	0.136	−14.103	−14.107
GPVC NLS	−20.436	22.235 (5)	−0.919	−1.209	0.048	−20.438	−20.440
LS GPVC	−15.361	22.807 (15)	−0.674	−0.902	0.165	−15.364	−15.368
NLS GPVC	−14.829	21.853 (1)	−0.679	−0.892	0.134	−14.832	−14.837
LS GPVC LS	−17.991	24.031 (38)	−0.749	−0.995	0.226	−17.996	−18.004
NLS GPVC NLS	−20.471	22.244 (6)	−0.920	−1.210	0.048	−20.472	−20.474
RPVC	−15.076	23.561 (27)	−0.640	−0.849	0.203	−15.080	−15.086
RPCV LS	−14.841	23.612 (32)	−0.629	−0.837	0.201	−14.844	−14.850
RPVC NLS	−23.341	23.711 (34)	−0.984	−1.289	0.134	−23.344	−23.349
LS RPVC	−18.340	23.935 (37)	−0.766	−1.017	0.226	−18.345	−18.353
NLS RPVC	−26.862	26.877 (46)	−0.999	−1.331	0.178	−26.868	−26.876
LS RPCV LS	−17.991	24.031 (38)	−0.749	−0.995	0.226	−17.996	−18.004
NLS RPVC NLS	−25.379	24.937 (45)	−1.018	−1.338	0.140	−25.383	−25.388

Table 8. Annualised performance measures: AV, SD, IR, SR and TO stand for the average, standard deviation, information ratio, Sortino's ratio and turnover of the out-of-sample MVP returns. AV_{20bp}^{net} and AV_{50bp}^{net} stand for the average out-of-sample MVP return net of transaction costs considering 20 and 50 basis-points, respectively. Period July 2009 to November 2017. The shaded cells denote the top five for each criterion. Weights are rebalanced on a monthly basis considering short-selling constraints.

	AV	SD	IR	SR	TO	$\mathbf{AV_{20bp}^{net}}$	$\mathbf{AV_{50bp}^{net}}$
1/N	13.130	16.057 (47)	0.818	1.148	-	-	-
CCC	12.592	10.935 (21)	1.152	1.666	0.333	12.583	12.569
CCC LS	12.200	10.873 (17)	1.122	1.628	0.342	12.191	12.178
CCC NLS	12.038	10.852 (15)	1.109	1.600	0.310	12.029	12.017
LS CCC	12.455	10.936 (22)	1.139	1.648	0.340	12.446	12.432
NLS CCC	12.466	10.894 (18)	1.144	1.656	0.333	12.457	12.443
LS CCC LS	11.992	10.932 (20)	1.097	1.592	0.336	11.983	11.970
NLS CCC NLS	12.656	10.945 (23)	1.156	1.679	0.340	12.646	12.633
DCC	11.729	10.801 (11)	1.086	1.554	0.336	11.720	11.706
DCC LS	11.873	10.761 (7)	1.103	1.590	0.340	11.864	11.850
DCC NLS	10.560	10.716 (3)	0.985	1.402	0.315	10.551	10.538
LS DCC	11.714	10.800 (10)	1.085	1.551	0.333	11.705	11.691
NLS DCC	11.701	10.801 (12)	1.083	1.549	0.333	11.692	11.678
LS DCC LS	11.845	10.761 (6)	1.101	1.585	0.340	11.835	11.821
NLS DCC NLS	10.534	10.714 (2)	0.983	1.397	0.312	10.525	10.512
RM2006	11.197	10.716 (4)	1.045	1.476	0.271	11.189	11.177
RM2006 LS	11.987	10.531 (1)	1.138	1.627	0.279	11.978	11.966
RM2006 NLS	10.943	10.776 (8)	1.016	1.442	0.291	10.935	10.923
RM1994	13.040	11.344 (37)	1.150	1.630	0.275	13.032	13.021
RM1994 LS	12.758	11.017 (27)	1.158	1.653	0.273	12.750	12.738
RM1994 NLS	12.229	11.068 (28)	1.105	1.566	0.252	12.222	12.211
DECO	11.054	11.293 (34)	0.979	1.415	0.321	11.045	11.032
DECO NLS	11.262	11.791 (45)	0.955	1.392	0.409	11.251	11.235
LS DECO	11.054	11.293 (34)	0.979	1.415	0.321	11.045	11.032
NLS DECO	11.055	11.293 (36)	0.979	1.415	0.321	11.046	11.033
NLS DECO NLS	11.262	11.791 (45)	0.955	1.392	0.409	11.251	11.235
OGARCH	10.576	10.959 (25)	0.965	1.377	0.103	10.573	10.569
OGARCH LS	9.694	10.852 (16)	0.893	1.281	0.120	9.691	9.686
OGARCH NLS	10.568	11.197 (33)	0.944	1.348	0.088	10.566	10.563
LS OGARCH	10.200	10.921 (19)	0.934	1.332	0.103	10.197	10.192
NLS OGARCH	10.580	10.958 (24)	0.966	1.377	0.103	10.577	10.573
LS OGARCH LS	9.853	10.847 (14)	0.908	1.304	0.115	9.850	9.845
NLS OGARCH NLS	10.581	11.197 (32)	0.945	1.350	0.088	10.579	10.576
GPVC	9.374	11.733 (43)	0.799	1.121	0.161	9.370	9.363
GPVC LS	9.194	11.126 (31)	0.826	1.168	0.124	9.191	9.186
GPVC NLS	9.425	11.598 (41)	0.813	1.146	0.145	9.421	9.416
LS GPVC	9.295	11.436 (38)	0.813	1.143	0.141	9.291	9.285
NLS GPVC	9.379	11.742 (44)	0.799	1.122	0.159	9.375	9.368
LS GPVC LS	9.617	10.737 (5)	0.896	1.283	0.180	9.612	9.605
NLS GPVC NLS	9.435	11.585 (40)	0.815	1.149	0.145	9.432	9.426
RPVC	11.163	11.486 (39)	0.972	1.376	0.268	11.155	11.144
RPCV LS	9.965	10.803 (13)	0.922	1.319	0.201	9.959	9.951
RPVC NLS	12.158	11.600 (42)	1.048	1.497	0.218	12.152	12.143
LS RPVC	10.150	11.125 (30)	0.912	1.291	0.239	10.143	10.133
NLS RPVC	10.847	11.091 (29)	0.978	1.388	0.212	10.841	10.833
LS RPCV LS	9.637	10.782 (9)	0.894	1.279	0.187	9.632	9.624
NLS RPVC NLS	11.130	10.964 (26)	1.015	1.451	0.189	11.125	11.118

As for the annualised average portfolio returns taking into account transaction costs, the procedures with the five largest values of AV_{20bp}^{net} and AV_{50bp}^{net} are the same procedures with the largest AV, except in some cases in the pre-crisis period, where one of five largest AV_{50bp}^{net} is obtained by the NLS-OGARCH-NLS procedure.

For each period, the five best methods in terms of information criteria are the same (except in Table 8, where four methods are the same). We omit the analysis in the crisis period because these criteria values are negative. Overall, for daily rebalancing, RiskMetrics based methods are among the best in the full and post-crisis periods, RPVC and RPVC-NLS are among the best in the full and pre-crisis periods, and NLS-OGARCH and LS-OGARCH are among the best in the pre-crisis period. For monthly rebalancing, some OGARCH-based methods are among the best in the pre-crisis and full periods, some CCC-based methods are among the best in the post-crisis and full periods, RM1994-LS is among the best for the post-crisis period, and RPVC is among the best for the full period.

The analysis of Tables 1–8 reveals that none of the methods is the best in all scenarios and the performance depends on the criterion, the period and the rebalancing strategy. In this sense, the analysis will focus on the full period (Tables 1 and 5) in order to account for periods with different volatility levels. When portfolios are rebalanced on a daily basis, we find that DCC-based methods are the best in terms of SD; RM2006-LS, RM2006-NL, RPVC and RPVC-NLS are the best in terms of {AV, AV^{net}_{20bp}, AV^{net}_{50bp}} and {IR, SR}, and some OGARCH-based are the best regarding TO. For monthly rebalanced portfolios, the best methods in terms of SD are DCC, LS-DCC, NLS-DCC, RM2006 and RM2006-LS, whereas the best performances in terms of {AV, AV^{net}_{20bp}, AV^{net}_{50bp}} and {IR, SR} are given by (RPVC, RPVC-NLS), (OGARCH-NLS,NLS-OGARCH-NLS) and CCC. In addition, the equal-weighted strategy is the second best in terms of AV, but the worst regarding SD, IR and SR criteria.

To show when the shrinkage method improves performance in terms of SD, the analysis is again focused on the full period (Tables 1 and 5). For daily and monthly portfolio rebalancing : shrinkage always improves the performance of the RM2004 and GPVC methods (except LS-GPVC for monthly rebalancing) whereas it always worsens the DCC method; linear shrinkage at the end improves RM2006; just linear/non-linear shrinkage at the beginning improves DECO; OGARCH-NLS and NLS-OGARCH-NLS improves OGARCH; LS-CCC improves CCC (as well as NLS-DCC for daily rebalancing). Additionally, for daily rebalancing, shrinkage always improves the performance of RPVC (except LS-GPVC), whereas for monthly rebalancing, linear shrinkage applied at the beginning and/or end improves RPVC. Nakagawa et al. (2018) also reports that in some cases the use of non-linear shrinkage on the unconditional covariance matrix of the devolatilised returns in the DCC model increases the standard deviation of the out-of-sample portfolio returns.

We now discuss the effect of shrinkage in terms of AV^{net}_{50bp}. For daily rebalancing, shrinkage improves the performance of the RM2006 and DECO methods, and worsens the performance of the DCC and RPVC methods. In addition, CCC-NLS is better than CCC, RM1994-NLS is better than RM1994, and LS-GPVC is better than GPVC. For monthly rebalancing, shrinkage does not improve the performance of the CCC, DCC, GPVC and RPVC methods. In addition, RM2006-LS is better than RM2006, RM1994-NLS is better than RM1994, DECO-NLS and NLS-DECO-NLS are better than DECO, and OGARCH-NLS and NLS-OGARCH-NLS are better than OGARCH.

Finally, we list next the main findings when short-selling is allowed for optimisation of the portfolio variance. A detailed analysis of these cases is given in the Supplementary Material. First, none of the methods is the best in all scenarios and the performance depends on the criterion, the sample period and the portfolio rebalancing scheme. Second, the analysis of the full period reveals that for daily rebalancing, DCC methods are the best regarding SD and are among the best in terms of IR and SR. RM1994-LS and RM2006-LS are the best according to AV, AV^{net}_{20bp}, AV^{net}_{50bp}, IR and SR. For monthly rebalancing, DCC-LS and LS-DCC-LS are among the best in terms of SD, RM2006-NLS is the best in terms of SD and is among the best regarding IR and SR. RM 1994 and RM1994-LS are the first and second best in terms of AV, AV^{net}_{20bp}, AV^{net}_{50bp} but are among the worst in terms of SD. Third, the analysis of the turnover and average net returns in the no short-sale constraints case must be carefully done. This is because since no limits are imposed on the weights of the portfolio, large turnover values can be obtained and consequently we can have a large loss (average return) but huge net gain (average net portfolio return taking into account transaction costs). Fourth, in many cases shrinkage improves the performance of the methods in terms of SD, and this improvement can be substantial. Fifth, the top-five models in terms of SD are the same in both restricted and unrestricted minimum variance portfolios for daily rebalancing, except in the crisis period.

4. Conclusions

The main conclusion of the paper is that none of the methods is the best in all scenarios and the performance depends on the criterion, the sample period, the portfolio rebalancing scheme and whether or not short-selling constraints are included in the portfolio optimisation process.

When short-selling constraints are included in the portfolio optimisation process, the main results can be summarised as follows. First, none of the methods is the best in all scenarios and the performance depends on the criterion, the sample period and the portfolio rebalancing scheme. Second, when considering the SD criterion, the five smallest SDs are obtained by the DCC based-methods, except in the crisis period, in which case, the five smallest SDs are spread among the DCC, OGARCH and GPVC based-methods. In the crisis-period, the smallest SDs are obtained by the GPVC procedure with the non-linear shrinkage applied to the one-step-ahead conditional covariance matrix. For portfolios rebalanced monthly, the smallest SDs are obtained by the RM2006-LS, NLS-DCC, NLS-GPVC and RM2006-LS procedures for the full, pre-crisis, crisis and post-crisis periods, respectively. Third, unlike Engle et al. (2017) and Nakagawa et al. (2018), we do not find that applying non-linear shrinkage to the unconditional correlation matrix of the devolatilised returns improves the performance of the portfolio in terms of SD when the DCC model is used, and this also happens when applied in other methods. It is important to point out that Engle et al. (2017) use portfolio of 1000 assets, Nakagawa et al. (2018) use portfolios of 100, 500 and 1000 assets and we use a portfolio with 174 assets.

When short-selling is allowed for optimisation of the portfolio variance, the main conclusions are: none of the methods is the best in all scenarios and the performance depends on the criterion, the sample period and the portfolio rebalancing scheme; in many cases shrinkage improves the performance of the methods in terms of SD and this improvement can be substantial; for daily rebalancing the top-five models in terms of SD are the same of those when short-selling constraints are imposed, except in the crisis period cases. Finally, focusing on the analysis of the full period cases we can say that overall the DCC and Riskmetrics-based methods are the best; and the analysis of the turnover and average net returns in the no short-selling constraints case should be carefully done.

Supplementary Materials: The following are available online at http://www.mdpi.com/2225-1146/7/2/19/s1, File: Covariance Prediction in Large Portfolio Allocation: Supplementary Material.

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Appendix A. Estimation Methods

Here we present the detailed list of the estimation methods implemented in the paper. The marginal variances in the CCC, DCC and DECO models were modelled by the GJR-(1,1) model (Glosten et al. 1993) and the parameters were estimated by quasi-maximum likelihood assuming a Student-t distribution. The volatility components in the GPVC and RPVC procedures were modelled by the GJR(1,1)-cDCC(1,1) model and its robust version proposed by Boudt et al. (2013) and

Laurent et al. (2016), respectively. The univariate variances in the OGARCH model were also modelled by the GJR-(1,1).

In the GPVC and RPVC procedures, the number of selected volatility components was estimated using criteria of Ahn and Horenstein (2013), Bai and Ng (2002) and Kaiser-Guttman Guttman (1954), and using the ratio estimator proposed by Lam and Yao (2012). Following these criteria and the suggestions in Trucíos et al. (2019), we use one volatility component in the GPVC procedure and four volatility components in the RPVC procedure.

The CCC, DCC, DECO, RM1994 and RM2006 procedures were implemented using the MFE Matlab Toolbox of Kevin Sheppard. The OGARCH, GPVC and RPVC procedures were implemented in R (R Core Team 2017) using the R packages *rugarch* of Ghalanos (2017), *Rcpp* of Eddelbuettel and François (2011) and *covRobust* of Wang et al. (2017). For the shrinkage procedures, we used the R packages *RiskPortfolios* (Ardia et al. 2018) and *nlshrink* (Ramprasad 2016) for the linear and non-linear shrinkage, respectively, coupled with the MATLAB toolbox QuEST (Ledoit and Wolf 2017) for the non-linear shrinkage and the MATLAB function *covCor*[5]. Whenever a program presented other options, we used the default options.

CCC based-methods

- CCC: Estimated by quasi-maximum likelihood.
- LS-CCC: Estimated as in CCC, but with the unconditional covariance matrix (Equation (4)) estimated using linear shrinkage.
- NLS-CCC: Estimated as in LS-CCC, but replacing linear by the non-linear shrinkage.
- CCC-LS: Estimated as in CCC, with the application of the linear shrinkage to the one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.
- CCC-NLS: Estimated as in CCC-LS, but replacing linear by non-linear shrinkage.
- LS-CCC-LS: Estimated as in LS-CCC, with the application of non-linear shrinkage to the one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.
- NLS-CCC-NLS: Estimated as in NLS-CCC, with the application of non-linear shrinkage to the one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.

DCC based-methods

- DCC: Estimated by composite likelihood (Pakel et al. 2014) using consecutive pairs.
- LS-DCC: Estimated as in DCC, but with the unconditional covariance matrix of the devolatilised returns ($\mathbf{C}$ in Equation (7)) estimated using linear shrinkage.
- NLS-DCC: Estimated as in LS-DCC, but replacing linear by non-linear shrinkage.
- DCC-LS: Estimated as in DCC, with the application of linear shrinkage to the one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.
- DCC-NLS: Estimated as in DCC-LS, but replacing linear by non-linear shrinkage.
- LS-DCC-LS: Estimated as in LS-DCC, with the application of linear shrinkage to the one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.
- NLS-DCC-NLS: Estimated as in NLS-DCC, with the application of non-linear shrinkage to the one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.

DECO based-methods

- DECO: Estimated using a single block.
- LS-DECO: Estimated as in DECO, but the unconditional covariance matrix of the devolatilised returns is estimated using linear shrinkage.

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[5] Available at www.econ.uzh.ch/en/people/faculty/wolf/publications.

- NLS-DECO: Estimated as in LS-DECO, but replacing linear by non-linear shrinkage.
- DECO-NLS: Estimated as in DECO-LS, but non-linear shrinkage is applied to the one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.
- NLS-DECO-NLS: Estimated as in NLS-DECO model, but with non-linear shrinkage applied to the $\mathbf{H}_{T+1}$ and linear shrinkage towards the equicorrelation matrix

Because in the DECO model the estimated unconditional covariance matrix and $\mathbf{H}_{T+1}$ are already equicorrelated there is no sense in using linear shrinkage towards the equicorrelation matrix, since it has no effect.

RiskMetrics based-methods

- RM1994: RM1994 method.
- RM1994-LS: Estimated as in RM1994 with linear shrinkage applied to the one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.
- RM1994-NLS: Estimated as in RM1994-LS but replacing linear by non-linear shrinkage.
- RM2006[6]: RM2006 method (Zumbach 2007).
- RM2006-LS: Estimated as in RM2006 with linear shrinkage applied to the one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.
- RM2006-NLS: Estimated as in RM2006-LS but replacing linear by non-linear shrinkage.

OGARCH based-methods

- OGARCH: The OGARCH model considers $k = N$ components.
- LS-OGARCH: Estimated as in OGARCH, but the unconditional covariance matrix used in the spectral decomposition is estimated using linear shrinkage.
- NLS-OGARCH: Estimated as in LS-OGARCH, but replacing linear by non-linear shrinkage.
- OGARCH-LS: Estimated as in OGARCH with the linear shrinkage applied to the one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.
- OGARCH-NLS: Estimated as in OGARCH-LS, but replacing linear by non-linear shrinkage.
- LS-OGARCH-LS: Estimated as in LS-OGARCH, but linear shrinkage is applied to the predicted one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.
- NLS-OGARCH-NLS: Estimated as in NLS-OGARCH, but non-linear shrinkage is applied to the predicted one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.

GPVC based-methods

- GPVC: The GPVC procedure considers $k = 1$ volatility component, as explained later. We use $g = 5$ as in Li et al. (2016).
- LS-GPVC: Estimated as in the GPVC model with the unconditional covariance matrix $\hat{\mathbf{\Sigma}}$ in Equation (17) estimated using linear shrinkage.
- NLS-GPVC: Estimated as in LS-GPVC, but replacing linear by non-linear shrinkage.
- GPVC-LS: Estimated as in GPVC with linear shrinkage applied to the one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.
- GPVC-NLS: Estimated as in GPVC-LS, but replacing linear by non-linear shrinkage.
- LS-GPVC-LS: Estimated as in LS-GPVC with linear shrinkage applied to the predicted one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.

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[6] This method was implemented using the MFE Matlab Toolbox of Kevin Sheppard with the default options. An R implementation of the same procedure can be found in Trucios (2017).

- NLS-GPVC-NLS: Estimated as in NLS-GPVC with non-linear shrinkage applied to the predicted one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.

RPVC based-methods

- RPVC: The RPVC procedure considers $k = 4$ volatility components, as explained later. We use $g = 5$ as in Li et al. (2016) and c as in Trucíos et al. (2019).
- LS-RPVC: Estimated as in RPVC, but linear shrinkage is applied to the robust unconditional covariance matrix $\hat{\Sigma}^R$ used in Equation (18).
- NLS-RPVC: Estimated as in LS-RPVC, but replacing linear by non-linear shrinkage.
- RPVC-LS: Estimated as in RPVC with linear shrinkage applied to the one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.
- RPVC-NLS: Estimated as in RPVC-LS, but replacing linear by non-linear shrinkage.
- LS-RPVC-LS: Estimated as in LS-RPVC with the linear shrinkage applied to the predicted one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.
- NLS-RPVC-NLS: Estimated as in NLS-RPVC with non-linear shrinkage applied to the predicted one-step-ahead conditional covariance matrix $\mathbf{H}_{T+1}$.

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